Parallelizing Programs

- Goal: speed up programs using multiple processors/cores

(Sequential) Matrix Multiplication

```
double A[n][n], B[n][n], C[n][n]  // assume n x n
for i = 0 to n-1
   for j = 0 to n-1
      double sum = 0.0
      for k = 0 to n-1
         sum += A[i][k] * B[k][j]
      C[i][j] = sum
```

Question: how can this program be parallelized?

Steps to parallelization

- First: find parallelism
  - Concerned about what can *legally* execute in parallel
  - At this stage, expose as much parallelism as possible
  - Partitioning can be based on data structures or by function

Note: other steps are architecture dependent

Finding Parallelism in Matrix Multiplication

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Finding Parallelism in Matrix Multiplication

- Can we parallelize the inner loop?
  - No, because *sum* would be written concurrently
- Can we parallelize the outer loops?
  - Yes, because the read and write sets are independent for each iteration (i,j)
    - Read set for process (i,j) is *sum*, *A[i][k=0:n-1], B[k=0:n-1][j]*
    - Write set for process (i,j) is *sum*, *C[i][j]*
  - Note: we have the option to parallelize just one of these loops

Terminology

- *co* statement: creates parallelism

```
co i := 0 to n-1
    Body
oc
```

- Meaning: *n* instances of body are created and executed concurrently until the end of the *co* (i.e., at the *oc*)
- Implementation: fork *n* threads, join them at the *oc*

Need to understand what processes/threads are!

Processes

- History: OS had to coordinate many activities
  - Example: deal with multiple users (each running multiple programs), incoming network data, I/O interrupts
- Solution: Define a model that makes complexity easier to manage
  - Process (thread) model

What's a process?

- Informally: program in execution
- Process encapsulates a physical processor
  - everything needed to run a program
    - code ("text")
    - registers (PC, SP, general purpose)
    - stack
    - data (global variables or dynamically allocated)
    - files
- NOTE: a process is sequential

Examples of Processes

- Shell: creates a process to execute command

  lectura:> ls foo

  (shell creates process that executes “ls”)

  lectura:> ls foo & cat bar & more

  (shell creates three processes, one per command)

- OS: creates a process to manage printer
  - process executes code such as:

    wait for data to come into system buffer

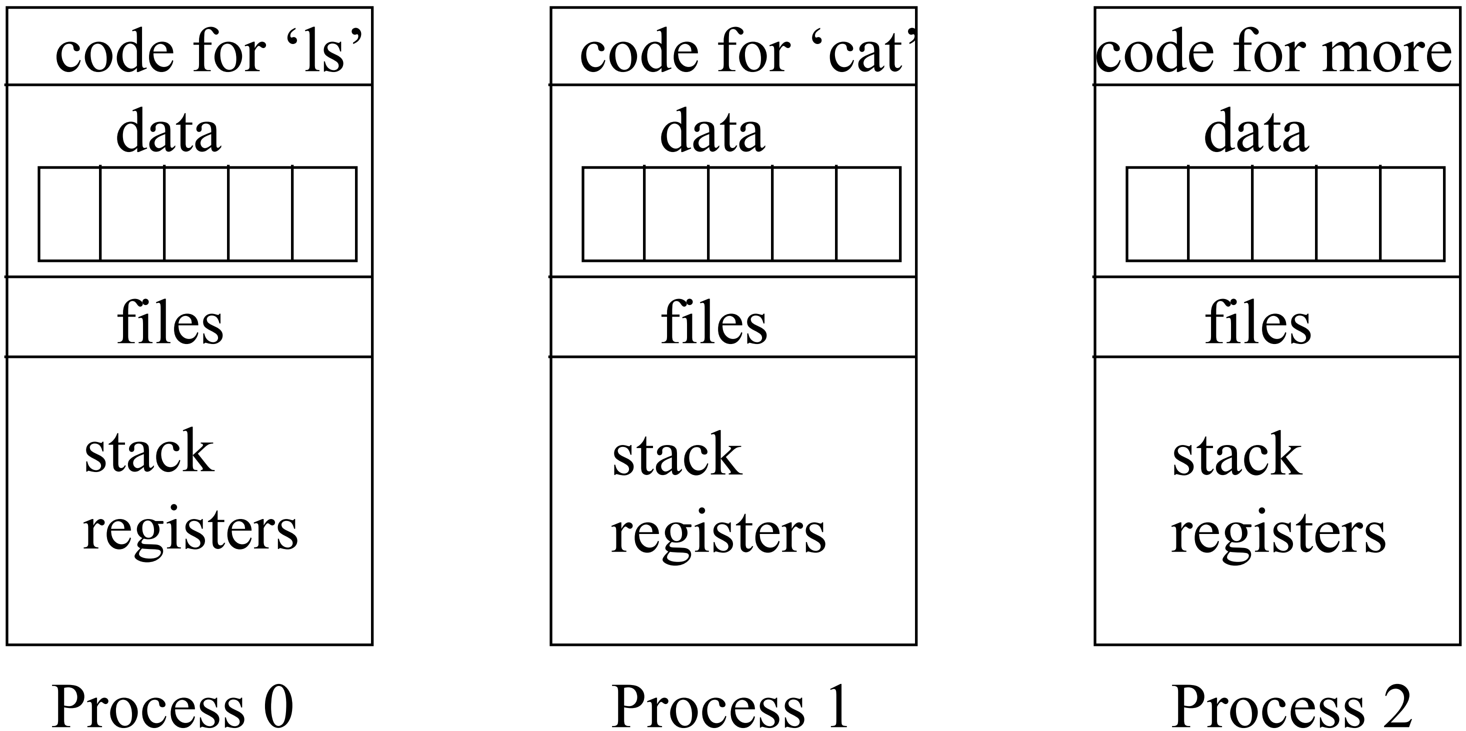
    move data to printer buffer

Creating a Process

- Must somehow specify code, data, files, stack, registers
- Ex: UNIX
  - Use the fork( ) system call to create a process
  - Makes an **exact** duplicate of the current process
    - (returns 0 to indicate child process)
  - Typically exec( ) is run on the child

We will not be doing this (systems programming)

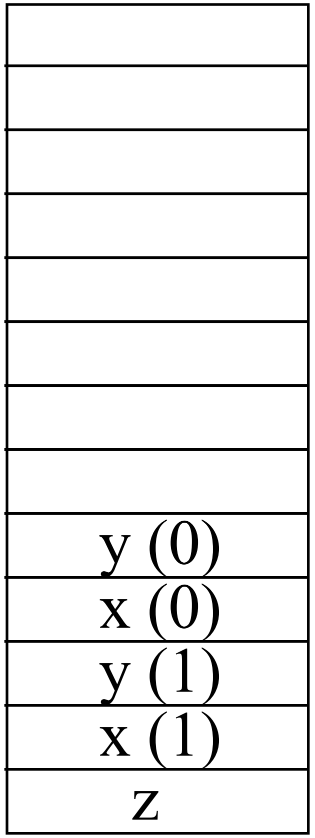
Example of Three Processes



OS switches between the three processes (“multiprogramming”)

Review: Run-time Stack

```
A(int x) {
    int y = x;
    if (x == 0) return;
    else return A(y-1) + 1;
}
B( ) {
    int z;
    A(1);
}
```



Decomposing a Process

- Process: everything needed to run a program
- Consists of:
  - Thread(s)
  - Address space

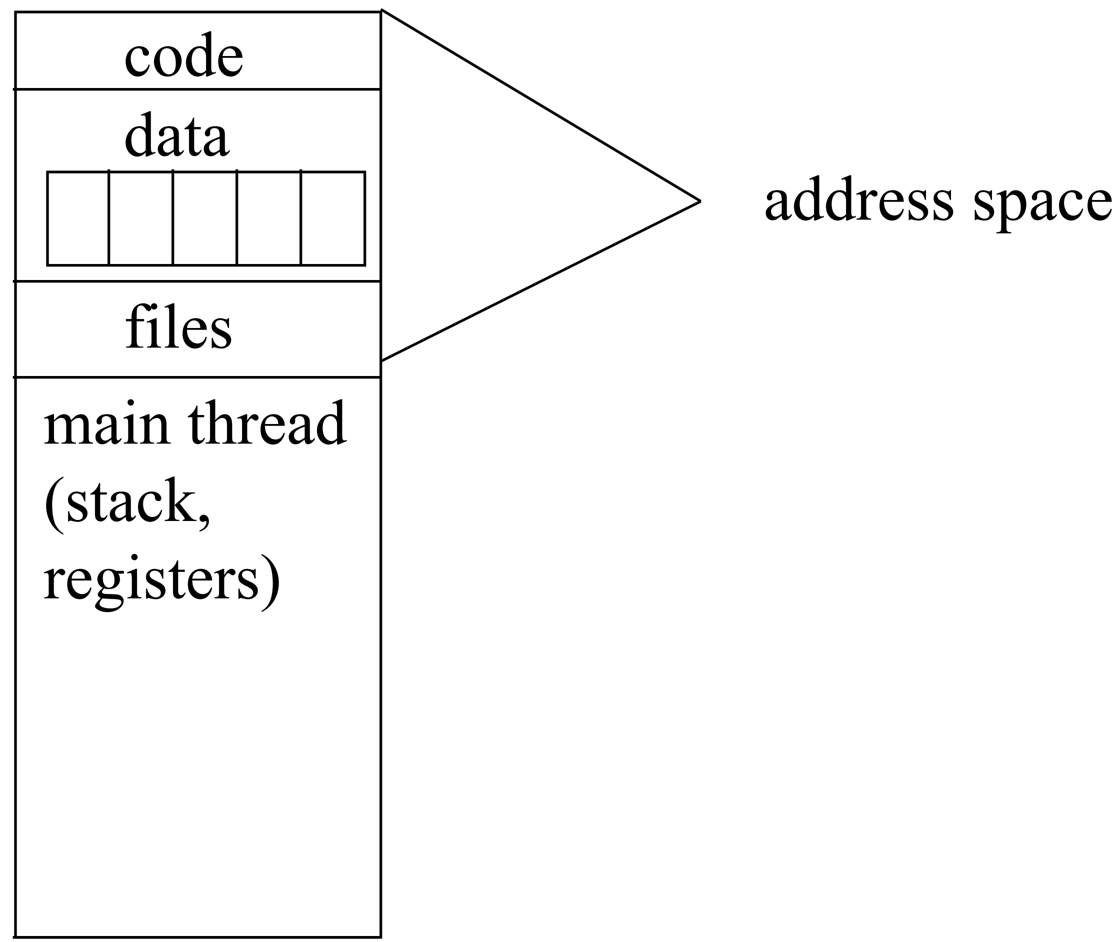
Thread

- Sequential stream of execution
- More concretely:
  - program counter (PC)
  - register set
  - stack
- Sometimes called lightweight process

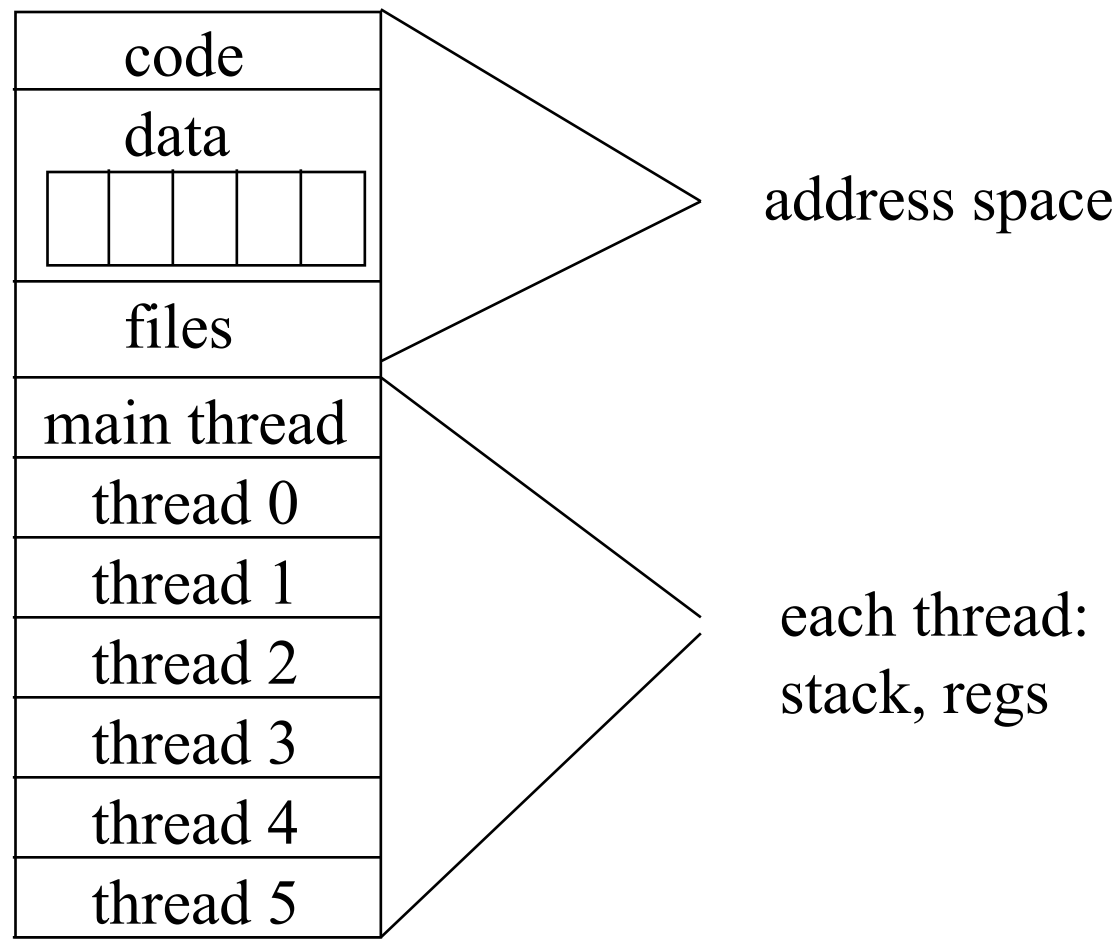
Address Space

- Consists of:
  - code
  - data
  - open files
- Address space can have > 1 thread
  - threads share code, data, files
  - threads have separate stacks, register set

One Thread, One Address Space



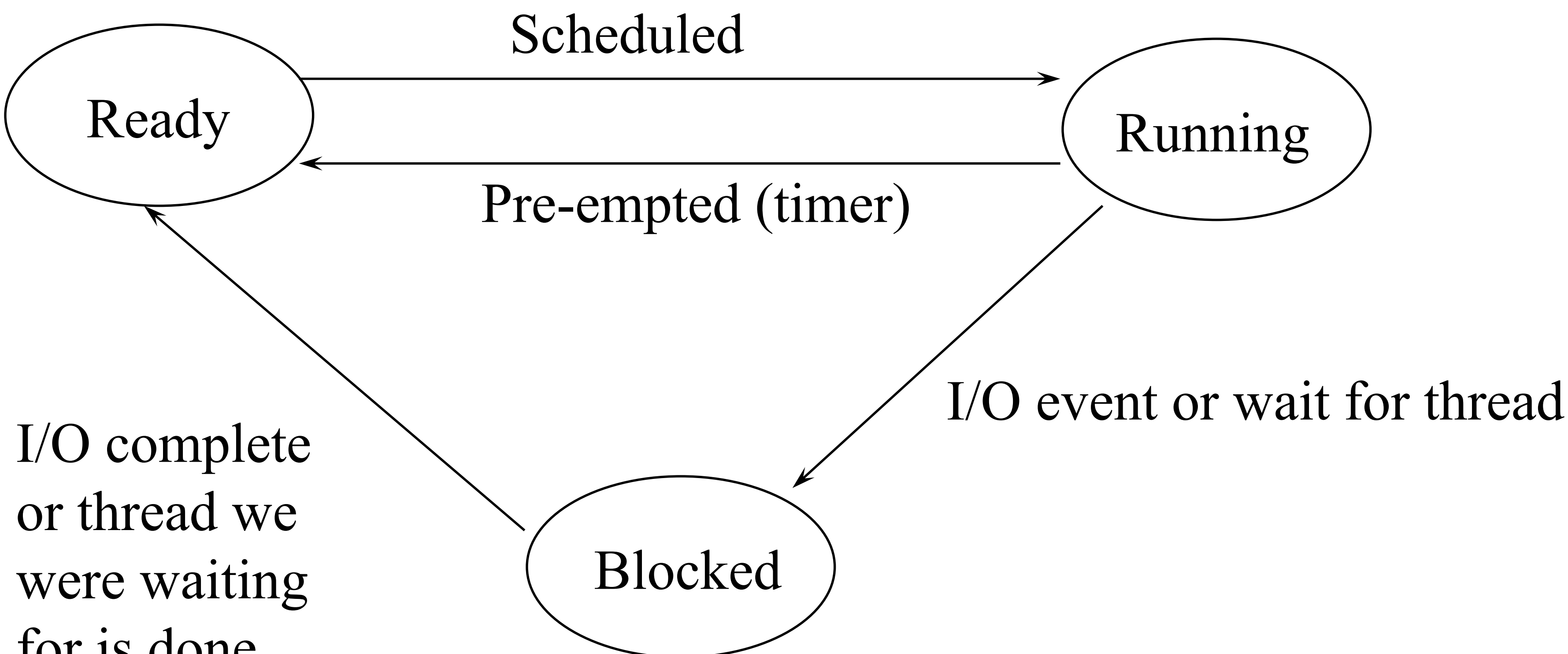
Many Threads, One Address Space



Thread States

- Ready
  - eligible to run, but another thread is running
- Running
  - using CPU
- Blocked
  - waiting for something to happen

Thread State Graph



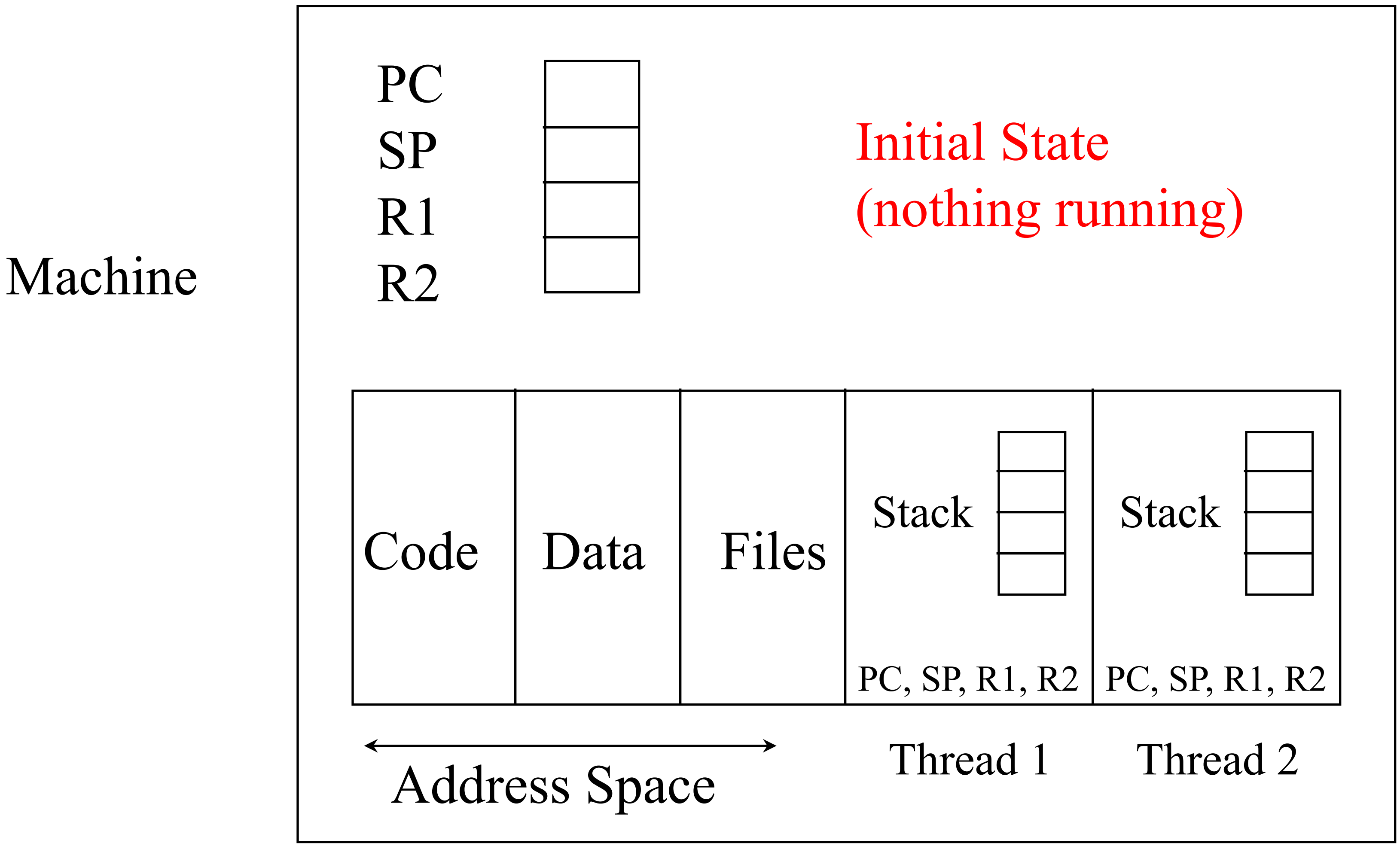
Scheduler

- Decides which thread to run
  - (from ready list only)
- Chooses from some algorithm
- From our point of view, the scheduler is something we cannot control
  - We have no idea which thread will be run, and our programs must not depend on execution order of two ready threads

Context Switching

- Switching between 2 threads
  - change PC to current instruction of new thread
    - **might need to restart old thread in the future**
  - must save exact state of first thread
- What must be saved?
  - registers (including PC and SP)
  - what about stack itself?

Multiple Threads, One Machine (Single Core)



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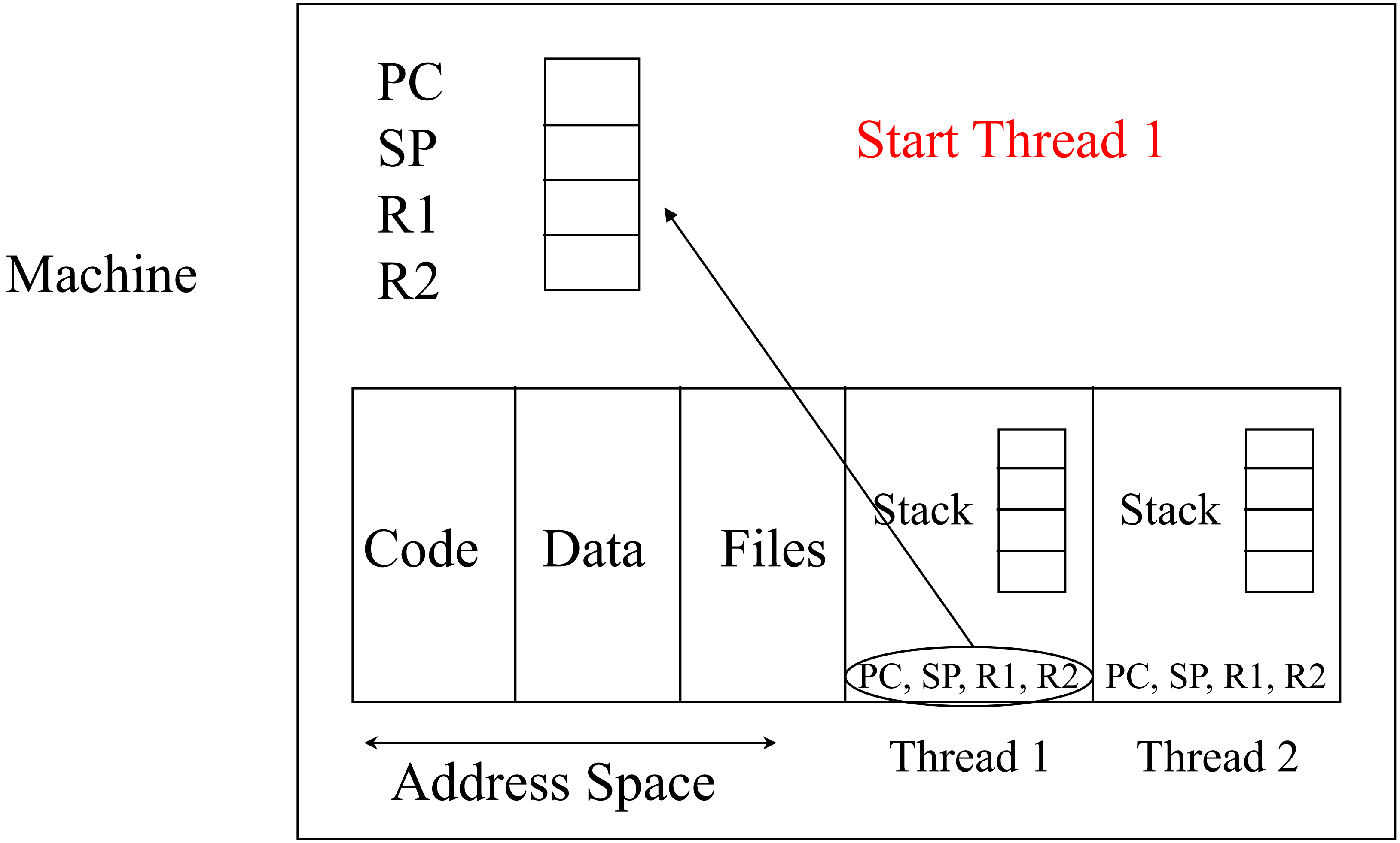

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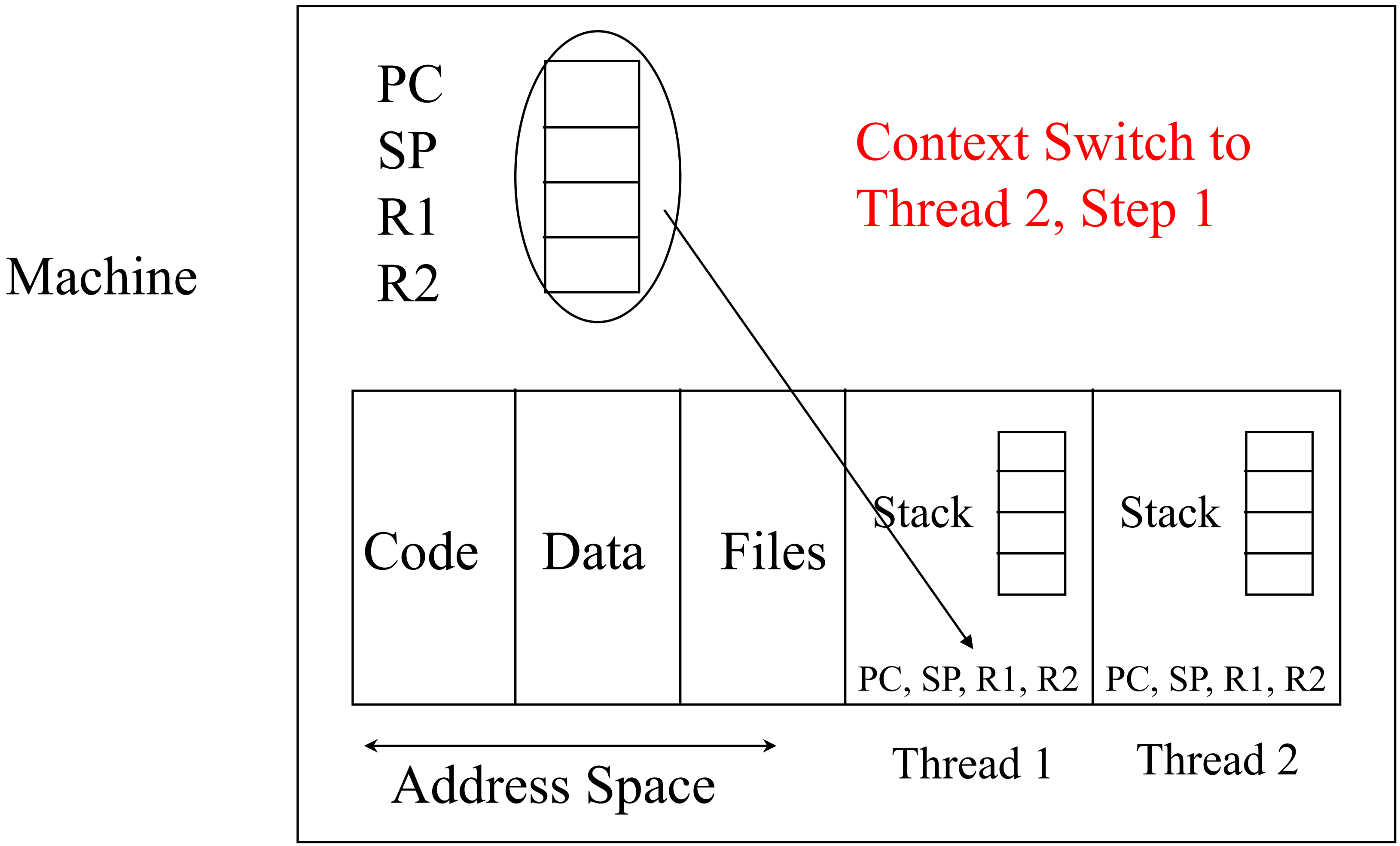

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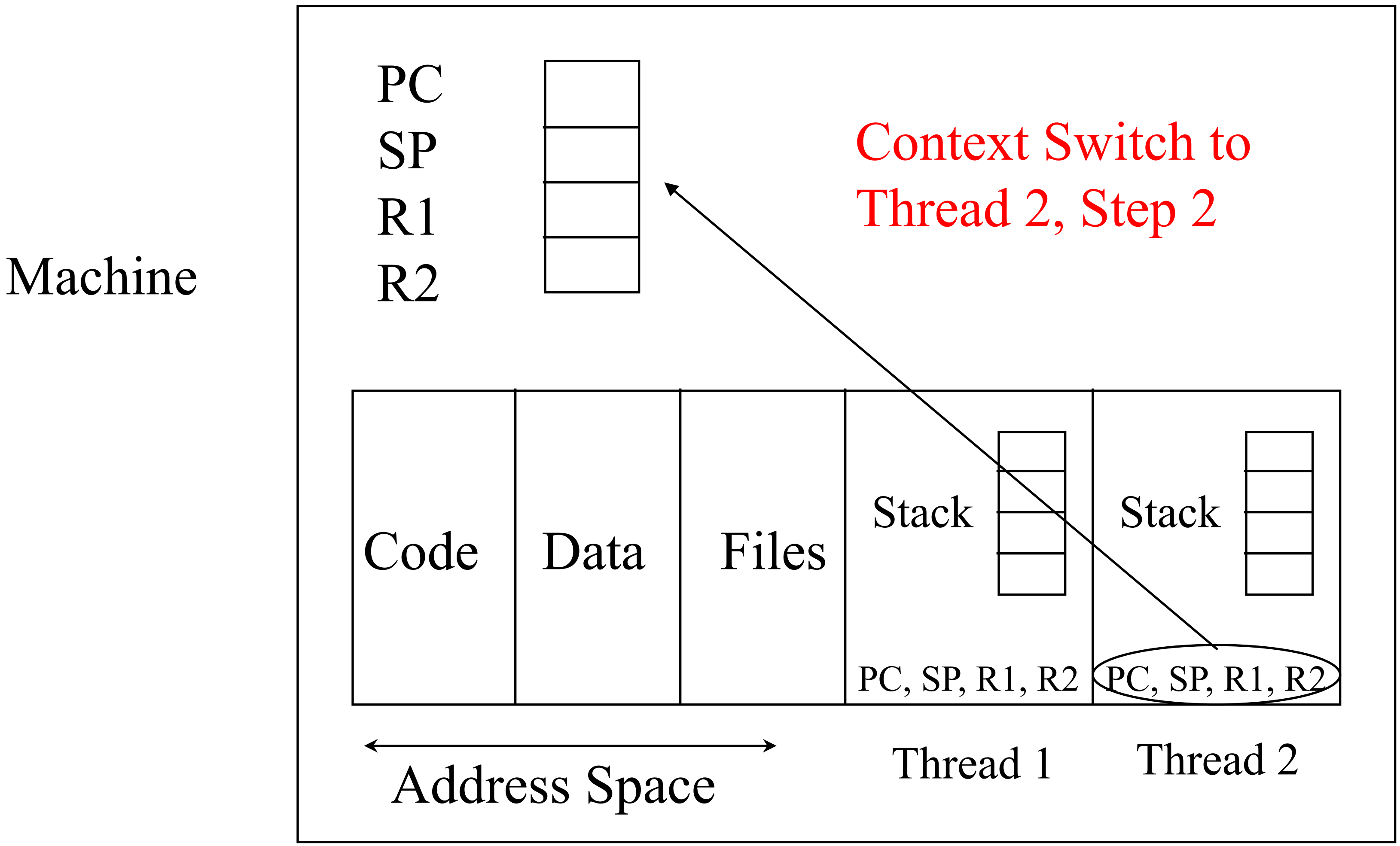

Why Save Registers?

- code for Thread 0

  foo( )

  x := x+1

  x := x*2

  Assembly code:

  R1 := R1 + 1 /* !! */

  R1 := R1 * 2

  Suppose context switch occurs after line "!!"

- code for Thread 1

  bar( )

  y := y+2

  y := y-3

  Assembly code:

  R1 := R1 + 2

  R1 := R1 - 3

Matrix Multiplication, n^2 threads

```
double A[n][n], B[n][n], C[n][n]  // assume n x n
co i = 0 to n-1  {
  co j = 0 to n-1  {
    double sum = 0.0
    for k = 0 to n-1
      sum += A[i][k] * B[k][j]
    C[i][j] = sum
  }
}
```

We already argued the two outer "for" loops were parallelizable

Steps to parallelization

- Second: control granularity
  - Must trade off advantages/disadvantages of fine-granularity
    - Advantages: better load balancing, better scalability
    - Disadvantages: process/thread overhead and communication
  - Combine small processes into larger ones to coarsen granularity
    - Try to keep the load balanced

Matrix Multiplication, *n* threads

```
double A[n][n], B[n][n], C[n][n]  // assume n x n
co i = 0 to n-1  {
  for j = 0 to n-1  {
    double sum = 0.0
    for k = 0 to n-1
      sum += A[i][k] * B[k][j]
    C[i][j] = sum
  }
}
```

This is plenty of parallelization if the number of cores is <= n

Matrix Multiplication, *p* threads

```
double A[n][n], B[n][n], C[n][n]  // assume n x n
co t = 0 to p-1  {
   startrow = t * n / p; endrow = (t+1) * n/p - 1
   for i = startrow to endrow
      for j = 0 to n-1  {
        double sum = 0.0
        for k = 0 to n-1
          sum += A[i][k] * B[k][j]
        C[i][j] = sum
      }
}
```

Steps to parallelization

- Third: distribute computation and data
  - Assign which processor does which computation
    - The co statement does *not* do this
  - If memory is distributed, decide which processor stores which data (why is this?)
    - Data can be replicated also
  - Goals: minimize communication and balance the computational workload
    - Often conflicting goals

Steps to parallelization

- Fourth: synchronize and/or communicate
  - If shared-memory machine, synchronize
    - Both mutual exclusion and sequence control
    - Locks, semaphores, condition variables, barriers, reductions
  - If distributed-memory machine, communicate
    - Message passing
    - Usually communication involves implicit synchronization

Parallel Matrix Multiplication---Distributed-Memory Version

```
process worker [i = 0 to p-1] {
  double A[n][n], B[n][n], C[n][n]  // wasting space!
  startrow = i * n / p; endrow = (i+1) * n/p – 1
  if (i == 0)  {
     for j = 1 to p-1 {
        sr= j * n / p; er = (j+1) * n/p – 1
        send A[sr:er][0:n-1], B[0:n-1][0:n-1] to process j
     }
  else
     receive A[startrow:endrow][0:n-1], B[0:n-1][0:n-1] from 0
```

Parallel Matrix Multiplication--- Distributed-Memory Version

```
for i = startrow to endrow
  for j = 0 to n-1 {
    double sum = 0.0
    for k = 0 to n-1
      sum += A[i][k] * B[k][j]
    C[i][j] = sum
  }
// here, need to send my piece back to master
// how do we do this?
} // end of process statement
```

Adaptive Quadrature: Recursive Sequential Program

```
 double f() {
  ....
}
double area(a, b)
  c := (a+b)/2
  compute area of each half and area of whole
  if (close)
    return area of whole
  else
    return area(a,c) + area (c,b)
```

Adaptive Quadrature: Recursive Parallel Program

```
double f() {
 ....
}
double area(a, b)
 c := (a+b)/2
 compute area of each half and area of whole
 if (close)
   return area of whole
 else
   co leftArea = area(a,c)  // rightArea = area (c,b) oc
   return leftArea + rightArea
```

Challenge with Adaptive Quadrature

- For efficiency, must control granularity (step 2)
  - Without such control, granularity will be too fine
  - Can stop thread creation after "enough" threads created
    - Hard in general, as do not want cores idle either
  - Thread implementation can perform work stealing
    - Idle cores take a thread and execute that thread, but care must be taken to avoid synchronization problems and/or efficiency problems

Steps to parallelization

- Fifth: assign processors to tasks (only if using task and data parallelism)
  - Must also know dependencies between tasks
  - Usually task parallelism used if limits of data parallelism are reached

Steps to parallelization

- Sixth: parallelism-specific optimizations
  - Examples: message aggregation, overlapping communication with computation

Steps to parallelization

- Seventh: acceleration
  - Find parts of code that can run on GPU/Xeon Phi/etc., and optimize those parts
  - Difficult and time consuming
    - But may be quite worth it